SEMESTER EXAMINATION, JANUARY, 2026

Course Name: B.Tech. **Semester: I[st]**

Paper Name: Engineering Mathematics-I **Paper Code: NBS-102**

Time: 3 Hrs. + 20 Minutes per hour extra time for V.I. and examinees with writer. **Max. Marks: 70**

समय - 3 घण्टे + 20 मिनट प्रति घण्टे अतिरिक्त दृष्टिबाधित एवं सहलेखक परीक्षार्थियों के लिए। अधिकतम अंक: 70

Instructions:

(i) The question paper consists of three Sections namely A, B and C. All Sections are compulsory.

(ii) Section-A – Each question carries 3 marks. All questions are compulsory.

(iii) Section-B – Answer any 5 out of 7 given questions. Each question carries 07 marks.

(iv) Section-C – Answer any 2 out of 3 given questions. Each question carries 10 marks.

निर्देश:

(i) प्रश्न-पत्र में तीन खण्ड अ, ब व स हैं। सभी खण्ड अनिवार्य हैं।

(ii) खण्ड-अ में प्रत्येक प्रश्न 03 अंकों का है। सभी प्रश्न अनिवार्य हैं।

(iii) खण्ड-ब में 07 प्रश्नों में से किन्हीं 05 प्रश्नों के उत्तर दीजिये। प्रत्येक प्रश्न 07 अंकों का है।

(iv) खण्ड-स में 03 प्रश्नों में से किन्हीं 02 प्रश्नों के उत्तर दीजिये। प्रत्येक प्रश्न 10 अंकों का है।

Section-A (खण्ड-अ)

Objective Questions (वस्तुनिष्ठ प्रश्न)

1. Answer all the following questions: (3×5=15)

निम्नलिखित सभी प्रश्न अनिवार्य हैं:

(i) The value of div $\frac{\vec{r}}{r^3}$ is:

(A) r (B) r^2

(C) 1 (D) 0

(ii) The value of $D^{10}(x+2)^7$ is:

(A) $\lfloor 10$ (B) $\lfloor 3$

(C) $\lfloor 7$ (D) 0

(iii) The vlaue of $\lfloor 3/2$ is:

(A) $\frac{1}{2}$ (B) $\sqrt{\pi}$

(C) $\frac{1}{2}\sqrt{\pi}$ (D) $\frac{1}{2}\sqrt{\pi}$

(iv) The characteristics roots of the matrix $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$ is:

(A) 5, 3, 2

(B) 1, 3, 2

(C) 1, 3, 3

(D) NOT

(v) The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3}+4-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}=0$ is:

(A) 3

(B) 2

(C) 2/3

(D) 3/2

Section-B (खण्ड-ब)

Short Answer Questions (लघुउत्तरीय प्रश्न)

2. Answer any five of the following questions: (7×5=35)

निम्नलिखित में से किन्हीं पाँच प्रश्नों के उत्तर दीजिये:

(i) State and prove Lagrange's Mean Value Theorem. Find the value of C by Lagrange's Mean Value Theorem if:

$f(x)=(x-1)(x-2)(x-3)$ in $[0, 4]$

(ii) Find the Rank of Matrix $c=\begin{bmatrix}2 & 1 & 0 & -1\\ 0 & 1 & 3 & 2\\ 1 & 0 & 2 & 3\end{bmatrix}$ by Normal Form.

and

Use Elementary transformation to find the inverse of matrix $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix}$.

(iii) Determine the consent a, b, c so that

$\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotation. Hence find the scalar function ϕ such that $\vec{F}=grad\phi$.

(iv) Solve $\left(D^3 + a^2 D\right) y = \sin ax$, $D = \frac{d}{dx}$ and evalaute $\int_0^1 \int_0^{x^2} e^{y/x} dx dy$.

(v) State and prove Rolle's theorem and verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1, 3]$.

(vi) State and prove Euler's theorem on Homogeneous function. Verify Euler's theorem for the function $U = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$.

(vii) Using Green's theorem evaluate $\int_c \left(x^2 y dx + x^2 dy\right)$ wehre c is boundary described counter clockwise of the Triangle with vertices (0, 0), (1, 0) (1, 1).

Section-C (खण्ड-स)

Descriptive Questions (विवरणात्मक प्रश्न)

3. Answer any two of the following questions: (10×2=20)

निम्नलिखित में से किन्हीं दो प्रश्नों के उत्तर दीजिये:

(i) Verify Gauss's Divergence theorem for $\vec{F} = x^2 \hat{i} + z \hat{j} + yz \hat{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1$ and $z = 0, z = 1$.

(ii) State Leibnitz's theorem, if $y = \tan^{-1}\left(\frac{a+x}{a-x}\right)$ prove that

$\left(x^2 + a^2\right) y_{n+2} + 2(n+1) x y_{n+1} + n(n+1) y_n = 0$ and if $y = e^x e^{2x}$ then find $(y_n)_0$.

(iii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Verify Cayley-Hamilton theorem and hence prove that:

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + 1 = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

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